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Analyzing Longitudinal Data using Multilevel Regression and Latent Growth Curve Analysis

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Abstract

Este trabajo investiga las diferencias entre la regresión multinivel estándar y el modelo de ecuaciones estructurales en relación al análisis de curvas de crecimiento. El modelo básico de curva de crecimiento tiene la misma especificación en ambos marcos, pero en muchos casos el modelado de ecuaciones estructurales es más flexible que el análisis de regresión multinivel. Esta flexibilidad implica la integración de la estructura factorial de la variable de medidas repetidas, estimando coeficientes para investigar la forma de las curvas de crecimiento, estructuras residuales alternativas, datos perdidos en variables predictoras y extensiones a modelos estructurales más complejos. El análisis de regresión multinivel, por otra parte, es más flexible al incorporar niveles más elevados dentro del modelo así como en las posibilidades de analizar datos con ocasiones vairables entre sujetos. No obstante, la distinción entre regresión multinivel y el análisis de curvas de crecimiento latente es actualmente difusa, y puede ser sólo una cuestión de tiempo el que los dos enfoques se fusionen entre sí. Hasta tal momento, este trabajo puede ayudar a facilitar la elección entre el análisis de regresión multinivel y el modelado de curvas de crecimiento latente. PALABRAS CLAVE: *Análisis de regresión multinivel, modelos de ecuaciones estructurales, análisis de curvas de crecimiento.*

Abstract

This paper investigates the differences between the standard multilevel regression and structural equation modeling framework regarding growth curve analysis. The basic growth curve model has the same specification in both frameworks, but in many instanc s structural equation modeling is more flexible than multilevel regression analysis. This flexibility concerns the integration of the factorial structure of the repeatedly measured variable, estimating basis function coefficients to investigate the shape of the growth curves, alternative residual structures, missing data on predictor variables, and extensions to larger structural models. Multilevel regression analysis, on the other hand, is more flexible in incorporating higher levels into the model and n the possibilities in analyzing data with varying occasions between subjects. However, the distinction between multilevel regression and latent growth curve analysis is now blurring, and it may only be a question of time before the two approaches will have merged into one another. Until that time, this paper may help to facilitate the choice between multilevel regression analysis and latent growth curve modeling. KEY WORDS: *Multilevel regression, Structural equation modeling, growth curve analysis*

Introduction

Longitudinal data originating from a panel design are common in the social and educational sciences. A wide array of statistical models is available for the analysis of panel data. In recent years, methods that take a growth curve perspective have become in fashion. Such growth curve models provide a way to account efficiently for

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the dependency caused by the fact that the same subjects have been assessed repeatedly. Other names for essentially the same model are: random-effects model (Laird & Ware, 1982), hierarchical model (Bryk & Raudenbush, 1987), random coefficients model (de Leeuw & Kreft, 1986), and mixed model (Longford, 1987). Several distinct techniques are available for analyses of this kind of model. In this paper we focus on two such techniques for the analysis of longitudinal data: multilevel regression (MLR) analysis (Bryk & Raudenbush, 1987, 1992; Goldstein, 1986, 1987, 1995), and latent growth curve (LGC) analysis (McArdle, 1986, 1988; Meredith & Tisak, 1990; Willett & Sayer, 1994).

Recent years show an increasing amount of applications of both longitudinal MLR and LGC analysis; see for instance Chan, Ramey, Ramey and Schmitt (2000), Garst, Frese and Molenaar (2000), Li, Duncan, Duncan, McAuley, Chaumeton and Harmer (2001), Muthén and Khoo (1998), Plewis (2000), and Raudenbush and Chan (1992; 1993). Several reasons exist for the current popularity of these techniques. On the one hand powerful software packages have become available for specifying and analyzing these longitudinal models (SAS Proc Mixed—Littell, Milliken, Stroup & Wolfinger, 1996; M*plus*—Muthén & Muthén, 1998; LISREL8.52—Jöreskog & Sörbom, 2002; Amos4.0—Arbuckle, 1999; MLwiN1.10—Rasbash, Browne, Healy, Cameron, & Charlton, 2000.; HLM5—Bryk, Raudenbush & Congdon, 1999). On the other hand, there is a growing amount of methodological literature in the form of tutorials and specialized papers dealing with longitudinal MLR analysis and LGC analysis (for example, see Chou, Bentler & Pentz, 1998; Duncan, Duncan, Strycker, Li & Alpert, 1999; Little, Schnabel & Baumert, 2000; Rovine & Molenaar, 1998, 2000; Singer, 1998). However, probably the most important reason for the popularity of these longitudinal growth models is their apparent elegance in representing both collective and individual change as a function of time.

The two approaches are highly similar. If they are used to represent the same set of longitudinal data and assumptions, their models yield identical estimates of the relevant parameters (Chou, Bentler & Pentz , 1998; Hox, 2000; MacCallum, Kim, Malarkey & Kiecolt-Glaser, 1997). This is not surprising because both approaches share the same objectives and have a similar representation. Although the different assumptions underlying structural equation modeling (SEM) and MLR analysis make them not comparable in general, SEM is comparable to MLR analysis when growth curves are studied (Chou, Bentler & Pentz, p.252). Differences appear in the possibilities in which the growth model can be extended, and in the ease in which such extensions can be specified in the available software.

Applied researchers are often confused about the differences and similarities between the two approaches to growth curve modeling. The purpose of this paper is to alert researchers to selected analytical issues that should be considered in the decision to apply one of these approaches to growth curve modeling, and to clarify the differences and similarities. That is, we focus explicitly on standard MLR and LGC analysis because these are approaches that are commonly available in the field of psychological and educational research. New programs like GLLAMM (Rabe-Hesketh, Pickles & Skrondal, 2001) and advanced extensions of older multilevel and SEM programs are now beginning to bridge the gap between the two approaches. However, these new programs and extensions require a high standard of technical knowledge, and are not yet in common use.

To make matters concrete, we shall refer throughout this paper to a hypothetical study in which data on the language acquisition of 300 children were collected during primary school at 4 consecutive occasions. Furthermore, data were collected on the children's gender and intelligence, as well as, on each occasion a measure of their emotional well-being. The interest of the study is in whether there is growth in language acquisition, and whether there are differences between the children concerning their growth curves. Given the interindividual differences in the growth curves, the study wants to investigate whether intelligence explains (part of) the interindividual variation in the growth curves and whether emotional well-being can be used to explain the time specific deviations from the mean growth curve. A final goal of the study is to investigate whether the growth in language acquisition (of their mother tongue) in primary education can be used to predict the achievement of foreign language acquisition at the first year of secondary education. The covariance matrix and means vector are presented in Table 1.

Table 1
Estimated covariance matrix and means vector

	y_1	y_2	y_3	y_4	x_1	x_2	x_3	x_4	z	w	means
y_1	1.581										9.827
y_2	1.275	4.152									11.723
y_3	1.519	4.910	8.903								13.655
y_4	1.772	6.832	11.264	17.500							15.647
x_1	.991	-.082	-.264	-.329	2.170						-.107
x_2	.162	1.441	.167	.181	.133	2.560					-.137
x_3	.071	-.125	1.204	-.273	-.064	-.090	2.345				-.070
x_4	.075	.230	.459	1.975	.027	-.038	.095	2.243			-.024
z	.341	1.278	2.096	3.010	-.084	.068	.007	.055	.956		-.036
w	.653	2.419	3.986	5.681	-.201	.180	.147	.431	1.167	2.805	4.297

Note: y=language acquisition, x=emotional well-being, z=intelligence, and w=foreign language acquisition.

Traditional multilevel regression and latent growth curve analysis

Longitudinal MLR analysis is based on a hierarchical linear regression model; LGC analysis on structural equation modeling. Both MLR and LGC analysis incorporate the factor 'time' explicitly. Within the MLR framework time is modeled as an independent variable at the lowest level, the individual is defined at the second level, and explanatory variables can be modeled at all existing levels. The interindividual differences in the parameters describing the growth curve are modeled as random effects. The LGC approach adopts a latent variable view with the dimension time incorporated explicitly in the specification of the latent variables. The parameters of the individual curves are modeled as latent variables, i.e. initial level[2] and linear growth rate, with a covariance and mean structure. The latent variables in LGC analysis correspond to the random effects in MLR analysis, and this makes it possible to specify exactly the same model as a LGC or MLR model. If this is done, exactly the same parameter estimates will emerge.

Since MLR and LGC analysis assume each individual's growth pattern to be represented by a unique curve, both approaches can be subsumed under the general term 'growth curve analysis'. Both make it possible to investigate questions about intra- and interidividual differences in developmental change. A detailed description of both approaches is beyond the scope of this paper. We refer to Raudenbush and

[2] Although the term 'level' might be a more appropriate name for this latent variable (see Stoel & van den Wittenboer, 2003), it will be confusing in combination with the standard multilevel terminology. Therefore we will use the substitute term 'initial level', with the remark that the first measurement occasion does not always correspond to the true origin of the growth process.

Bryk (2002), Longford (1993), Goldstein (1995), Van der Leeden (1998), Hox (2000; 2002), Snijders and Bosker (1999), and Browne and Rashbash (2002) for a descriptio of MLR analysis and statistical derivations; and to Meredith and Tisak (1990), Willett and Sayer (1994), MacCallum et al. (1997) for a description of LGC analysis.

In a simple growth curve model, there are no important differences in the setup of the model. A general multilevel equation for a 2-level growth model is (see also Hox, 2000, p.19):

$$\begin{aligned} y_{ti} &= \pi_{0i} + \pi_{1i} T_{ti} + \pi_2 x_{ti} + e_{ti} \\ \pi_{01} &= \beta_{00} + \beta_{01} z_i + r_{0i} \\ \pi_{1i} &= \beta_{10} + \beta_{11} z_i + r_{1i} \\ e_{ti} &\sim N(0, \sigma_e^2) \end{aligned} \tag{1}$$

$$\begin{bmatrix} r_{0i} \\ r_{1i} \end{bmatrix} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_v), \boldsymbol{\Sigma}_v = \begin{bmatrix} \sigma_{v0}^2 & \\ \sigma_{v01}^2 & \sigma_{v1}^2 \end{bmatrix}$$

where T_{ti} is a variable denoting the measurement occasion [0, 1, 2, 3]. The initial level and linear shape for each individual subject are expressed by the coefficients (π_{0i} and π_{1i} with expectations β_{00} and β_{10}, and random deviations r_{0i} and r_{1i}; π_2 represents the effect of the time-varying covariate x_{ti}; β_{00} and β_{11} represent respectively the effects of the time-invariant predictor z_i on the initial level and linear shape, and finally, e_{ti} is a residual at the measurement level.

The same model can easily be expressed as a LGC model in the usual SEM language:

$$\begin{aligned} y_{ti} &= \lambda_{0t}\eta_{0i} + \lambda_{1t}\eta_{1i} + \gamma_{2t} x_{ti} + \epsilon_{ti} \\ \eta_{0i} &= \nu_0 + \gamma_0 z_i + \zeta_{0i} \\ \eta_{1i} &= \nu_1 + \gamma_1 z_i + \zeta_{1i} \end{aligned}$$

Here, y_{ti} also represents the measure of individual i on occasion t. However, the initial level and linear shape are now represented by the latent factors, η_{0i} and η_{1i}, with expectations ν_0 and ν_1, and random departures, ζ_{0i} and ζ_{1i}, respectively. Time is introduced by constraining the factor loadings, λ_{0t} and λ_{1t}, to known values of respectively $[1,1,1,1]$ and $[0,1,2,3]$; γ_{2t} represents the effect of the time-varying covariate x_{ti}; γ_0 and γ_1 are the effects of the time-invariant covariate on the initial level and linear shape. The variances of ζ_{0i} and ζ_{1i}, and their covariance are represented by respectively, Ψ_{00}, Ψ_{11}, and Ψ_{01}.

The estimation of both MLR and LGC models is usually done by the Maximum Likelihood method. The Maximum Likelihood (ML) method is an iterative estimation procedure, which produces estimates for the population parameters that maximize the probability of observing the data given the model (cf. Eliason, 1993). ML estimation proceeds by maximizing the likelihood function. Given the relatively large sample size, maximum likelihood estimation is expected to produce estimates that are asymptotically efficient and consistent. In this paper Full Maximum Likelihood

(FML) estimation is used for the MLR models, and ML estimation for the LGC. Both FML and ML include the variance components and regression coefficients simultaneously in the estimation, and can be regarded as equivalent.

Alternative estimation methods like Restricted Maximum Likelihood (RML), Markov Chain Monte Carlo (MCMC), and Bootstrapping methods (cf. Mooney & Duval, 1993), are beyond the scope of this paper. These alternative methods have their own peculiarities and may sometimes be preferred to ML estimation. Simulation based MCMC methods, for example, do give better estimates for some problems and can be applied to more complicated models where there is no equivalent iterative procedure available at the moment Browne & Rashbash, 2002). We refer to Goldstein (1995), Browne & Rashbash (2002), Loehlin (1987) and Bollen (1989) for a description of these alternative estimation methods.

As an illustration of the equivalence of the two techniques, we will now present a growth curve analysis of the data on language acquisition using both MLR (using MLwiN 1.10) and LGC analysis (using M*plus* 1.04). The data used in this example consist of the scores on language acquisition of the 300 children, measured on 4 occasions (y_{ti}), the repeatedly assessed measure of emotional well-being (x_{ti}), and the measure of intelligence (z_i); the covariates represent standardized variables.

Analyzing the data using both the MLR and LGC approach with Maximum Likelihood estimation leads to the parameter estimates presented in Table 2. The first column of Table 2 presents the relevant parameters; the second and third columns show the parameter estimates of respectively MLR, and LGC analysis. The residuals, e_{ti}, as well as the effect of emotional well-being at each occasion, π_2 respectively γ_2, are hypothesized to be equal over time. The LGC model, as a special type of structural equation model is often represented by a path diagram. The model and the parameter estimates of then LGC model are, therefore, also represented in Figure 1.

Table 2
Maximum Likelihood estimates of the parameters of Equation 1, using multilevel regression and latent growth curve analysis

Parameter	MLR	LGC
Fixed part		
β_{00}; ν_0	9.891 (.056)	9.891 (.056)
β_{10}; ν_1	1.955 (.048)	1.955 (.048)
β_{01}; γ_0	.398 (.058)	.398 (.058)
β_{11}; γ_1	.903 (.049)	.903 (.049)
β_2; γ_{2t}	.549 (.013)	.549 (.013)
Random part		
σ_e^2; σ_ϵ^2	.251 (.014)	.251 (.014)
$\sigma_{\nu 0}^2$; Ψ_{00}	.775 (.078)	.775 (.078)
$\sigma_{\nu 1}^2$; Ψ_{11}	.642 (.057)	.642 (.057)
$\sigma_{\nu 01}^2$; Ψ_{01}	.003 (.047)	.003 (.047)

Note: Standard errors are given in parentheses. The Chi-square test of model fit for the LGC model: $\chi^2(25) = 37.84(p = .95)$; RMSEA = .00. For the MLR model: -2*loglikelihood = 3346.114.

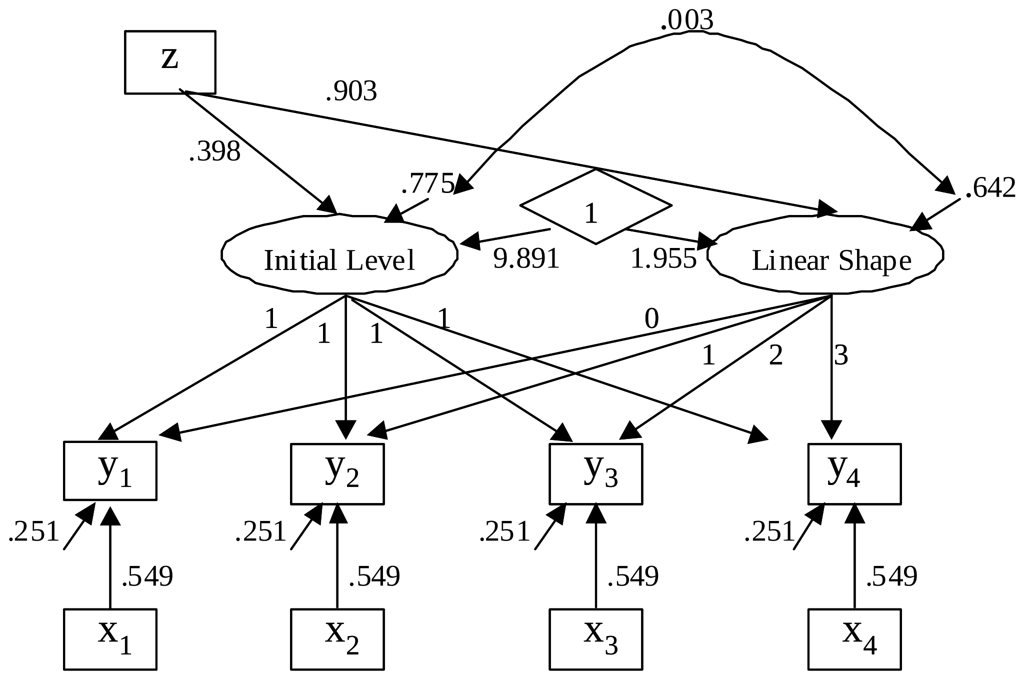

Figure 1. Graphic representation of the growth model with a time-varying and time-invariant covariate

As one can see in Table 2 the parameter estimates are the same and, as a consequence, both approaches would lead to the same substantive conclusions. The conclusions can be summarized as follows. After controlling for the effect of the covariates, a mean growth curve emerges with an initial level of 9.89 and a growth rate of 1.96. The significant variation between the subjects around these mean values implies that subjects start their growth process at different values and grow subsequently with different rates. The correlation between initial level and growth rate is zero. In other words, the initial level has no predictive value for the growth rate. Intelligence has a positive effect on both the initial level and growth rate, leading to the conclusion that more intelligent children show a higher score at the first measurement occasion and a greater increase in language acquisition than children with lower intelligence. Emotional well-being explains the time specific deviations from the mean growth curve. That is, children with a higher emotional well-being at a specific time point show a higher score on language acquisition than is predicted by their growth curve.

The most striking difference between these models is the way time is introduced. In the multilevel model time is introduced as a fixed explanatory variable, whereas in the LGC model it is introduced via the factor loadings. So, in the longitudinal MLR model an additional variable is added, and in the LGC model the factor loadings for the repeatedly measured variable are constrained in such a way that they represent time. The consequence of this is that with reference to the basic growth curve model, MLR is essentially a univariate approach, with time points treated as observations of the same variable, whereas the LGC model is essentially a multivariate approach, with each time point treated as a separate variable[3]. This is an important distinction because it will help us later on to explain why:

- MLR analysis does not allow factor loadings to be estimated (Section 2.1 and 2.2).
- MLR models usually assume constant parameters over time, including residual variances and the effects of time-varying covariates (Section 2.3).
- LGC models allow more flexible specification of residual correlation structures (Section 2.3).
- MLR analysis is better at handling missing responses and different numbers of measurement occasions (Section 2.5).

That fact that MLR analysis is essentially a univariate approach while LGC analysis can be considered a multivariate approach is nicely illustrated by looking at the way the data are set up. Table 3 and 4 present the data of 2 randomly chosen subjects from the language acquisition study for respectively MLR and LGC analysis.

Table 3
Data format for the longitudinal MLR model for two randomly selected subjects

variable	y	x	Z	t
subject				
1	10.36	.83	.44	0
1	11.64	1.15	.44	1
1	12.99	.17	.44	2
1	14.31	-2.13	.44	3
2	10.52	-1.02	-.06	0
2	12.82	2.67	-.06	1
2	12.00	-1.59	-.06	2
2	16.51	2.96	-.06	3

Table 3 shows that the values of language acquisition (y) are treated as scores on just one variable while the same holds for emotional well-being (x). The scores of intelligence (z) are represented by one variable with each score repeated 4 times for each subject. Time (t) is treated as an observed variable that will enter the model as a fixed covariate. The data setup for LGC analysis in Table 4 shows that each variable measured at a specific occasion is treated as a separate variable. Time is not modeled as an observed variable, but instead via constraints of the basis function (i.e. the factor loadings).

[3]That the MLR approach to growth curve analysis is essentially univariate in nature does not imply that the model cannot be extended to the multivariate case. By using dummy variables, as suggested by Goldstein (1995), a multivariate growth curve model an be estimated in which simultaneously multiple outcome variables are modeled. However, since the multiple outcome variables are treated as realizations of a single outcome variable (MacCallum et al., 1997, p. 222; MacCallum & Kim, 2000, p.56), the model remains essentially univariate but it is given a multivariate interpretation. We will further discuss the opportunities of the multivariate MLR model in Section 2.1.

Table 4
Data format for the LGC model for the two randomly selected subjects

variable / subject	y_0	y_1	y_2	y_3	x_0	x_1	x_2	x_3	z
1	10.36	11.64	12.99	14.31	.83	1.15	.17	-2.13	.44
2	10.52	12.82	12.00	16.51	-1.02	2.67	-1.59	2.96	-.06

In this paper we will show that differences arise if the growth model is extended or if certain assumptions are violated. We will discuss instances in which MLR and LGC analysis possess distinctive properties. In some instances MLR analysis looks more suitable, or may even be required; in others LGC analysis must be preferred. We will describe such instances, and discuss the differences between the two approaches.

Extensions of the measurement model

Often, multiple indicators of the underlying construct are available. SEM offers a number of possibilities for modeling multiple indicators using a common factor measurement model. Illustrative examples of a LGC model with multiple indicators can be found in Garst et al. (2000), and Hancock, Kuo, and Lawrence (2001). The multiple indicator growth model, or curve-of-factors model (McArdle, 1988), is a higher order factor model. It merges a common factor model for the multiple indicators at each occasion with a growth curve model of the common factor scores over time (Duncan et al., 1999). In other words, the common variation in the multiple indicators is accounted for by the first-order factors, while the second-order factors serve to explain the mean and covariance structure of the first-order factors. Using similar restrictions as in the single indicator growth model, the second-order factors can be given the interpretation of initial level and linear shape. Thus, instead of analyzing the sum scores or parcels, as is the standard practice within MLR, the observed variables can be put directly into the analysis. A graphic presentation of a growth curve model with multiple indicators is presented in Figure 2.

The ability to model multiple indicators under a common factor is actually one of the main advantages of SEM in general, as is confirmed by the extensive literature on issues concerning common factors and structural models (e.g. Bollen, 1989). Whenever multiple indicators of an outcome are available, the possibilities of MLR analysis constitute only a subset of the potentials using LGC analysis.

If multiple indicators of a given construct are available at each occasion, SEM allows for an explicit test of the assumption of measurement invariance. This assumption underlies any longitudinal factor model and ensures a comparable definition of the latent construct over time (Garst et al. 2000; Hancock et al., 2001; Kenny & Campbell, 1989; Oort, 2001; Plewis, 1996). An explicit test of this assumption is necessary before any further analysis can be performed. In practice, the assumption of measurement invariance implies equal factor loadings and equal indicator intercepts across time for each repeatedly measured indicator. A violation of the assumption hinders the assessment of change because it will be confounded with change of the meaning of the construct over time. For an adequate assessment of change it is important that the construct under investigation does not have an altered content (i.e. meaning) over time.

Raudenbush, Rowan and Kang (1991) proposed a way to perform a confirmatory factor analysis within the MLR model. In this approach the indicators are modeled on a separate level together with the assumption of them being parallel measures (i.e. residual variances and factorloadings of the indicators within each factor are equal). We refer to Hox (2002) for a recent illustration of multilevel confirmatory factor analysis (also, see Li, Duncan, Harmer, Acock & Stoolmiller, 1998). The multiple-indicator growth model could, in principle, be analyzed by MLR using the method of Raudenbush et al. However, the restrictions on the factor structure are implicit in the model and cannot be relaxed, and the assumption of measurement invariance can, therefore, not be tested explicitly. Factor loadings and residual variances are assumed to be equal within each occasion, but no restrictions across time can be made. As a consequence, the solution provided by Raudenbush et al. (1991) is only a partial solution to the problem.

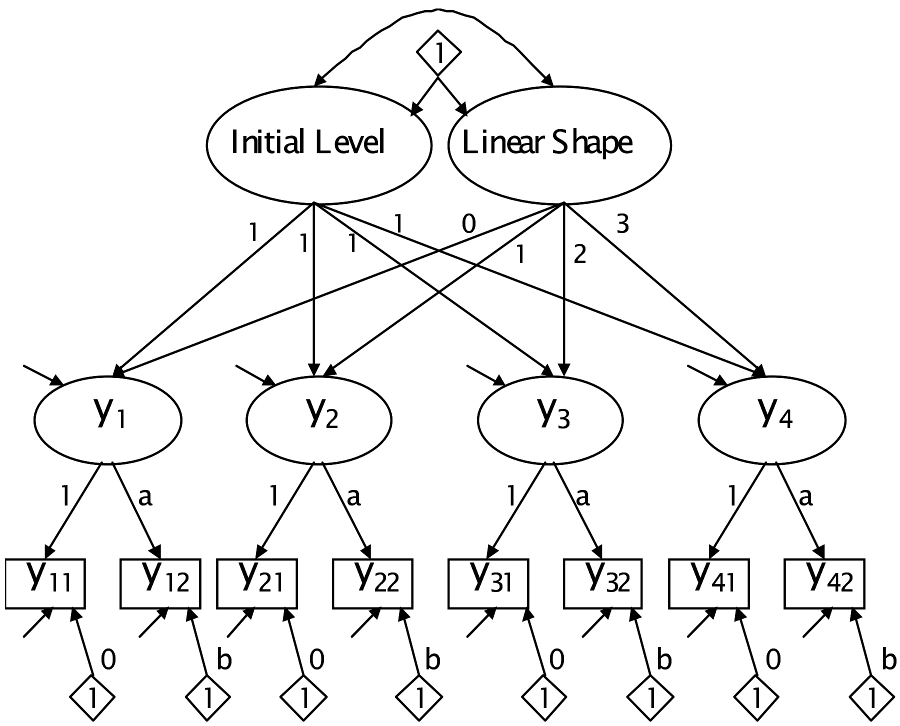

Figure 2. Graphic presentation of a LGC with full measurement invariance

Note: Intercepts of indicators are conceptualized as regression on a constant (See Hancock et al, 2001). Factor loadings and intercepts for y_{t1} are fixed to, respectively, 1.00 and zero prior to estimation; factor loadings of y_{t2} are constrained to be equal (a); intercepts of y_{t1} are fixed to zero; intercepts of y_{t2} are constrained to be equal (b).

The problem of measurement invariance can nicely be illustrated with the language acquisition data described in the previous section. Suppose that language acquisition was measured using two indicators at each of the four time points instead of just a single indicator. This can be analyzed in a LGC model using the two separate indicators instead of taking the average to create a single score at each time point as is standard practice in MLR analysis. In this section, the results of the LGC analysis on the individual indicators are presented. The model is presented in Figure 2. It is estimated with and without the measurement invariance constraints.

The chi-square difference test can be used to test the two models since the model with the constraints is nested within the model without the constraints. If the constraints lead to a significant deterioration of the model fit, it must be concluded that the assumption of measurement invariance is violated, and it must be concluded that the indicators can not be assumed to measure the same latent construct at each point in time.

The model without constraints for measurement invariance has the following overall fit measures: $\chi^2(19) = 33.52(p = .02)$; RMSEA = .050. Although the fit of this model is acceptable, a substantive interpretation may not be given to the parameter estimates since it is not sure if the repeatedly observed indicators measure the same latent variable on the same scale. Therefore, the model has to be estimated with the appropriate constraints for measurement invariance. This model has the following overall fit measures: $\chi^2(25) = 138.94(p = .00)$; RMSEA = .123. Comparing the unrestricted model with the restricted model gives a χ^2 difference of 138.94 - 33.52 = 105.42 with 25 - 19 = 6 degrees of freedom (p=.00): a significant deterioration in model fit. The assumption of measurement invariance is clearly violated in these data, and alternative ways of modeling the data must be explored.

An alternative that has not been used often for modeling multiple indicators is to model two separate growth curves for each repeatedly measured indicator in a multivariate LGC model. As noticed in Footnote 2, this model can also be estimated in the MLR framework. This state of affairs might give the impression that there does exist an adequate way of modeling multiple indicators with MLR analysis. This is not completely true because the aim of the multivariate growth curve model is to investigate questions regarding change in conceptually different constructs (e.g. Is growth in language acquisition related to growth in mathematical skills?), and not to combine multiple indicators that are assumed to measure the same concept. If multiple indicators of the same construct are analyzed in a multivariate growth curve model, interpretational problems may arise on how to combine the separate growth curves into one. Furthermore, if the assumption of measurement invariance appears to be violated, at least one of the indicators does not behave as expected a priori. In other words, the indicator cannot be regarded to measure the same construct at the different occasions, impeding the analysis of change. In this situation, the multivariate growth curve model merely masks the test of measurement invariance than that it solves it. However, the relationship between the multivariate growth curve model, and the multiple indicator growth curve model is interesting, and future research by the corresponding authors will be devoted to the comparison of the two approaches.

Since a multivariate growth curve model is essentially a combination of two univariate growth curve models in which the growth parameters are allowed to covary, it is not expected that differences in parameter estimates emerge (cf. MacCallum et al., 1997). A detailed illustration of the multivariate model is therefore not presented here. The interested reader is referred to the work of MacCallum et al. (1997) and MacCallum and Kim (2000) presenting multivariate growth curve models from both the LGC and MLR perspective.

An interesting, but intricate, feature of the analyses presented here is the generalization of the results of this section to the results of Section 2, where means of the two indicators were analyzed at each occasion. In a strict sense, the violation of the assumption of measurement invariance implies that analyzing the means, or any weighting of the two indicators, is not permitted. However, analyzing means, or scale scores, impedes the test of measurement invariance. In that sense, it adds to the current discussion on measurement invariance and 'item-parceling' (See Bandalos, 2002; Byrne, Shavelson & Muthén, 1998; Stoel, Van den Wittenboer & Hox, 2003), by demonstrating the danger of bluntly taking averages of indicators in structural equation modeling. For the ease of exposition, however, the analyses in the remaining of this paper are performed on single indicators, assuming they each accurately measure the construct of language acquisition.

Estimating the shape of the growth curve

The linear latent growth curve model is often too restricted to fit the data. A possible way out of this type of situation is to include one or more higher-order polynomial terms into the growth model to account for the nonlinear growth or development present in the data or theory, for instance, via a quadratic or cubic term in a polynomial growth curve model. It can be shown easily that the MLR model and the LGC model share the same features regarding the estimation of nonlinear growth curves by means of polynomials and that the same parameter estimates emerge (cf. MacCallum et al. 1997).

An alternative approach to the inclusion of higher-order polynomial terms is the "latent basis" approach, originating from the work of Rao (1958) and Tucker (1958), and introduced in SEM by McArdle (1986) and Meredith and Tisak (1990). In contrast to higher-order polynomial growth curve models, in which all coefficients of the basis functions (i.e. the factor loadings) are fixed to known values, the "latent basis" approach describes nonlinearity in the growth curves by estimating the basis function coefficients for the growth factor.

The issue to be discussed in this section also has to do with the estimation of factor loadings, and is therefore related to the issue discussed in the Section 1.3. The growth curve models discussed up to now all assume linear growth. The factor time is incorporated explicitly in the model, by constraining the factor loadings (LGC analysis) or by including time as an independent variable (MLR analysis). In most instances, the fixed values of the factor loadings, and the values of the time variable represent the occasions at which the subjects where measured. As described by McArdle (1988) and Meredith and Tisak (1990) within the LGC approach, it is possible to estimate a more general LGC model in which the factor loadings, or basis function values, for the growth rate are estimated. Thus, instead of constraining the basis function for the growth rate to e.g. $[0, 1, 2, \ldots, T]$, the basis function is set to $[0, 1, b_3, b_4, \ldots, b_T]$. In other words, the factor loadings b_3 to b_T are left free to be estimated, providing information on the shape of the growth curve. For purposes of identification, at least two basis coefficient values need to be fixed. The remaining values are estimated to provide information on the shape of the curve. Muthén and Khoo (1997) explain this as the estimation of the time scores. The essence is captured effectively with the following citation of Garst (2000, p.259). "Statistically, a linear model is still estimated, but the nonlinear interpretation emerges by relating the estimated time scores to the real time frame... Therefore, a new time frame is estimated and the transformation to the real time frame gives the nonlinear interpretation".

Although this model has a similar representation in MLR analysis (c.f. MacCallum et al., 1997), it cannot be estimated using this approach. As a univariate approach, the repeated measurements are treated as realizations of just one dependent variable, and time is represented by another independent variable. It is required that the values of the independent variable are known.

To illustrate the LGC model with an estimated basis function, suppose that a researcher wants to test if a nonlinear model provides a better fit to the language acquisition data than a linear growth curve model. A nonlinear growth curve model is estimated in which the last two basis function values are estimated. Table 5 presents the relevant parameter estimates; the estimates of the linear LGC model are included for comparison[4].

Table 5
Maximum Likelihood estimates of the linear LGC model and the LGC model with estimated basis function values

Parameter	linear LGC	LGC with estimated basis function x_t
Fixed part		
Initial level	9.804 (.07)	9.823 (.08)
Growth Rate	1.939 (.08)	1.916 (.10)
Random part		
σ_e^2; σ_ϵ^2	.951 (.06)	.948 (.06)
$\sigma_{\nu 0}^2$; Ψ_{00}	.873 (.13)	.848 (.13)
$\sigma_{\nu 1}^2$; Ψ_{11}	1.486 (.14)	1.452 (.17)
$\sigma_{\nu 01}^2$; Ψ_{01}	.340 (.09)	.348 (.09)
basis function	[0, 1, 2, 3]	[$\underline{0}$, $\underline{1}$, 1.989, 3.044]
CHISQ	$\chi^2(8) = 13.19(p = .10)$.	$\chi^2(6) = 11.54(p = .07)$.
RMSEA	.046	.055

Note: Standard errors are given in parentheses.

As can be seen from Table 5 the parameter estimates of the LGC model with estimated basis function are very close to the parameter estimates of the linear LGC. This is not so strange because the linear model already provided a good fit. However, the estimates are not identical. Judged from the estimated basis function values, there is a small amount of nonlinearity present in the latent growth curves. That is, the estimated factor loadings are not completely equivalent to the values of the measurement occasions. Note that the chi-square difference between the linear and the nonlinear model is not significant: $\chi^2(2) = 1.65$, $(p > .05)$, implying that the more parsimonious linear model should be preferred. The larger the deviation of the estimated basis function values from the values under linear growth (e.g. $[0, 1, 2, 3]$), the stronger is the nonlinearity of the latent growth curves, ultimately leading to a significant chi-square difference test statistic and rejection of the linear model.

An interesting aspect of the MLR model is that the value of the deviance (-2*log-likelihood) is the lowest when the values of the time variable, T_{ti}, are fixed to the values of the estimated basis function of the LGC model. Thus, in principle, it must

[4]The model is estimated without the time varying and time invariant covariate.

be possible to find these values by repeatedly estimating the MLR model while changing the values of the time variable T_{ti}. This has not been suggested elsewhere in the literature; on the contrary, some authors explicitly state that a growth curve model with an estimated basis function cannot be estimated using MLR analysis (e.g. MacCallum et al. 1997; Chou et al., 1998). As a preliminary illustration, the likelihood ratio test between the MLR model with the time variable fixed to $[0, 1, 2, 3]$ and the model with the time variable fixed to $[0, 1, 1.9893.044]$ is equal to the likelihood ratio test for the LGC model (4551.023 - 4549.374 = 1.65 with 2 degrees of freedom). This illustrates that this is indeed the optimal solution from a MLR perspective.

Alternative error structures and time-varying covariates

In the analysis of the language acquisition data, it was assumed that the residuals are homoscedastically and independently distributed over time and that the effects of emotional well-being, the time-varying covariate, are constant across time. Although they led to a good model fit for these data, these assumptions are too strict in many practical situations. Alternative structures for the level-1 residuals and different effects of the time-varying covariate at each occasion can be incorporated into the model in both MLR and LGC analysis. However, the approaches differ to some degree in the flexibility of incorporating the alternative structures for the residuals.

As a multivariate approach and a special case of the more general SEM, LGC analysis is very flexible regarding these issues. Each occasion is treated as a separate variable, and it is a natural extension of SEM to estimate each of the variances, and/or covariances. Moreover, since the time-varying covariate is treated as a different variable at each occasion, it is easy to estimate a separate effect at each occasion. As a matter of fact, LGC analysis automatically assumes different parameters for different variables unless additional constraints are specified. The effectiveness of a variety of reasonable error structures, and time-varying effects, can systematically be compared and the structure that is most appropriate for the particular problem adopted (Willett & Sayer, 1996).

Regarding the residuals, some software packages designed for MLR analysis support preprogrammed residual structures (e.g. SAS Proc Mixed). Other packages provide such structures in the form of macros, which can be downloaded from the Internet (e.g. MLwiN). The number of available covariance patterns is limited, however. In the standard setup, the MLR model does not allow for changing effects of the time-varying covariate. However, using interactions between the time-varying covariate with dummy variables for the occasions, it is possible to estimate different effects for the time-varying covariate at each occasion. Given the similarities between MLR and LGC analysis, it is expected that the same parameter estimates emerge. The equation for the MLR model with varying effect of the time-varying covariate is presented in the Appendix.

For the language acquisition data, the estimation of the growth curve model with different effects of the time-varying covariate gives, indeed, the same parameter estimates for the MLR model and the LGC model. In addition, the likelihood ratio test gives the value of 20.58 with 3 degrees of freedom in both approaches leading to a rejection of the model with constant effects. The effects of the time-varying covariate at the 4 occasions are respectively, .476 (.03), .510 (.02), .597 (.02), and

.636 (.03): the effect of well-being on time specific deviations from the growth curve seems to be getting stronger during primary education. In other words, in the course of time well-being becomes a better predictor of why children have a better, or worse, level of language acquisition than expected from their individual growth curve.

Three and multiple level models

Inclusion of more than two levels might pose problems within the SEM framework. Assume that the children from the language acquisition data were sampled from a sample of 30 classes. The data set thus exists of 3 hierarchical nested levels instead of two: the repeatedly measured outcome variable is assigned to first level, subjects are assigned to second level, and schools to highest level. Thus, measurements are nested within subjects, who are subsequently nested within schools. When the 3-level structure is ignored, the assumption of independently and identically distributed (i.i.d.) observations is violated. This is the classical argument for the use of multilevel models because children within the same classroom will be more similar than children from different classrooms. Unjustly ignoring the multilevel structure leads to underestimated standard errors, and thus to spuriously significant effects. The MLR model together with the available software is especially suited to estimate this type of 3 level models, or even models with more levels in the hierarchy[5].

Consider the linear growth model again, in which the repeatedly measured subjects are nested in classes. Typical questions to be answered with such a model are questions like: How does language acquisition change as a function of time? Does the pattern of change vary from subject to subject? Can this variation be explained using the time-varying and time-invariant covariates, respectively emotional well-being and intelligence? Are children within the same class more similar than between classes (i.e. is there intra-class correlation)? If so, does the pattern of change vary between classes? Can this variation be explained using class-level covariates (e.g. the number of children in one classroom)?

Questions like these can be investigated relatively straightforward using existing multilevel regression software. Incorporating extra levels in the hierarchy constitutes a natural extension, and thus poses no special problems. As mentioned above, special extensions have been developed as well for SEM, making it possible to estimate growth models with more than two levels Muthén (1994, 1997). Because the random effects are modeled as latent factors within LGC analysis, it is possible, indeed, to estimate both random intercepts and random slopes on the third level. Thus, a three level growth model (occasion nested in subjects, which are nested in classes) can fully be estimated within SEM, but the procedure is far more complex than within MLR analysis. Moreover, the setups for models with more than three levels become increasingly complex. However, the method of Muthén is limited to random intercepts on higher levels. Moreover, it uses limited information instead of full information maximum likelihood; as a result, standard errors tend to be biased downwards (Hox & Maas, 2001). Readers interested in a detailed enunciation of the method of Muthén are referred to Muthén (1997), or Hox (2002).

[5]E.g. MLwiN1.10 (Rashbash et al., 1998) can handle up to 50 Levels; HLM5 (Raudenbush, Bryk & Congdon, 1999) routinely up to 3.

Missing data

One of the advantages of the MLR analysis is its ability to handle missing data (Bryk & Raudenbush, 1992; Hox, 2000; Snijders, 1996). As a univariate technique, MLR analysis does not assume "time-structured data" (Bock, 1979), so that the number of measurement occasions and its spacing need not be the same for all individuals. Thus, the absence of measurements on a subject on one or more occasion poses no special problems; and/or the subjects may be measured at different occasions. In such instances, the time variable, as a fixed independent variable, will just have different and/or a different number of scores for the subjects. In an extreme case there may be many measurement occasions, but there may be just one observation at each occasion. This can be seen as an advantage of the MLR model. As a result, MLR analysis easily models longitudinal data with a less extreme pattern of observations, such as for example panel dropout. The ability of MLR analysis to easily handle cases with missing values on the outcome variable (i.e. missing occasions) is the natural result of the fact that MLR does not assume balanced data to begin with. MLR has no special provisions for incomplete data otherwise, so it cannot deal with missing values on the covariates.

LGC analysis as such, on the contrary, assumes time-structured data (fixed occasions). Special procedures are needed if the number and spacing of the measurement occasions varies. One possibility is to estimate the growth model using all available data of all cases using full-information maximum likelihood estimation (Muthén, Kaplan & Hollis, 1987; Wothke, 2000). This approach, also called raw data ML approach, constitutes a principled way to deal with incomplete data in any part of the data matrix. In brief, the procedure sorts the observations into different missing data patterns with all patterns subsequently being analyzed in a multiple group design with appropriate constraints across the groups. Thus, the same model is estimated for all groups, and the subjects with missing data are not removed from the analysis. At least four SEM packages provide such full-information maximum likelihood estimation (Amos, Mx32—Neale, 2002, Mplus and Lisrel 8.5). Similar to MLR full-information maximum likelihood does not make the restrictive assumption of Missing Completely At Random (MCAR), but instead it is based on the less restrictive assumption that the missing values are Missing At Random (MAR; Little & Rubin, 1987). MAR assumes that the missing values can be predicted from the available data. It is not our purpose here to give a broad explanation of this procedure and of the different estimation techniques. However, the advantage of MLR analysis concerning missing observations only holds for missing observations on the dependent variable. Whenever explanatory variables are missing, the growth model as such, cannot be estimated by MLR analysis. If an explanatory variable is missing, the usual treatment within MLR analysis is to remove the subject from the analysis by listwise deletion whereas the LGC model can still be estimated using the full-information maximum likelihood approach.

Extensions of the structural model

Frequently, research hypotheses are not restricted to the pattern of change in a single process, but they focus instead on simultaneously modeling the change in several outcome variables (c.f. cross-domain analysis, Willett & Sayer, 1996; MacCallum et al. 1997) or on modeling relationships between growth parameters and variables serving as outcomes of those parameters (e.g. Garst et al., 2000). In the language acquisition study, for example, the interest is in whether growth in language acquisition (of their mother tongue) in primary education, and intelligence can be used to predict the achievement of foreign language acquisition (w) at the first year of secondary education. As an illustration, Figure 3 presents the path diagram of this extended LGC model, with foreign language acquisition regressed on the growth rate and intelligence.

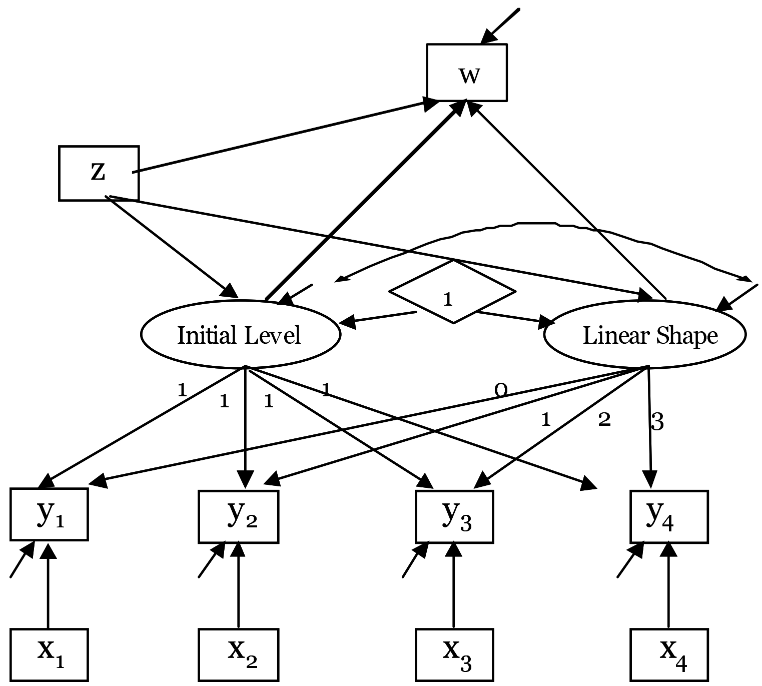

Figure 3. Graphic representation of the LGC model with the outcome variable foreign language acquisition.

A distinction between MLR analysis and LGC analysis relates to this kind of extensions of the "structural part" of the model. Compared to SEM, the multilevel approach is severely limited in modeling extensions of this kind. The current software merely allows the inclusion of measured predictors on all existing levels, and the estimation of the covariances between growth parameters in a multivariate model (e.g. MacCallum et al., 1997). Posterior Bayes estimates (see Raudenbush & Bryk, 2002) of the initial level and growth parameters can be computed, and be subsequently included as predictors in a separate multiple regression analysis[6]. This would, however, require the use of two software packages, but it can, nevertheless, have advantages in specific situations. SEM, on the other hand, is more flexible. It is possible to estimate all means and covariances associated with the latent growth parameters, or they can be modeled explicitly. As a consequence, SEM is better prepared

[6]Another option could be to estimate a multivariate model with the outcome variable(s) included. The covariance matrix of the growth parameters now includes also the outcome variable, and it can be analyzed separately within the SEM framework.

to estimate (1) (directional) interrelationships among several growth processes, (2) simultaneous and joint associations of these growth processes and covariates, and (3) mediated effects of covariates (see also Willett & Sayer, 1994). In other words, the growth curve model as modeled in SEM, can be part of a larger structural model, in which, for example, the effects of the latent initial level and linear shape factor on other variables can be modeled simultaneously, as was shown in Figure 3.

To demonstrate the inclusion of such an extended growth curve model, the model in Figure 3 is estimated for the language acquisition data. The parameter estimates for the growth part of the model are similar to those presented in Table 2 and Section 2.3. The extended model provides the following fit measures: $\chi^2(28) = 36.99, p = .12$; RMSEA = .033. The effects of respectively intelligence, the initial level, and the growth rate of language acquisition on foreign language acquisition are: .362 (.087), .385 (.068), and .783 (.070). Thus, the effect of intelligence on foreign language is significantly different from zero. In addition, both the initial level and the growth rate have an effect on foreign language acquisition in the first year of secondary education.

The model could also have been analyzed within the multilevel framework with one of the approaches mentioned above. Since just one outcome variable was used, a multiple regression analysis was performed with the measure of intelligence and the posterior Bayes estimates of the initial level and growth rate as predictor variables. The effects of the predictors on foreign language were, respectively: 1.22 (.056), .385 (.068), .781 (.071). These estimates are very similar to the estimates for the LGC model. Please note that the effect of intelligence on foreign language includes the indirect effects: 1.22 = .362 + (.385*.398) + (.781*.903). Measures of model fit are lacking in this approach.

Discussion

In this paper, differences between the standard MLR and SEM framework regarding growth curve analysis are investigated and illustrated. Although the basic growth model has the same specification in both frameworks, it turns out that in many instances, SEM is more flexible than MLR analysis. This flexibility concerns, (1) integration of the factorial structure of the repeatedly measured variable, (2) estimating basis function coefficients to investigate nonlinear growth curves, (3) incorporating alternative structures for the level-1 residuals, (4) analyzing data with missing values on predictor variables, and (5) incorporating the growth model in a larger structural model. It is illustrated how the multivariate perspective of LGC analysis explains some of its advantages with respect to the MLR analysis. Treating the repeated measurements as different variables results in a much more flexible approach than treating them as observations of just a single variable.

On the other hand, MLR analysis is much more flexible in incorporating higher levels into the model. For cross-sectional data it is, using current software, not possible to estimate a fully equivalent multilevel model using SEM. However, this flexibility is not as pronounced for a growth curve model that consists of only three levels (measurements, subjects, schools). It is straightforward to specify and estimate such a model using the method of Muthén (1994). The flexibility, and advantages, of MLR analysis are becoming more important when a model incorporates more than three levels. Another advantage of MLR analysis is that the number of measurement occasions and its spacing need not be the same for all subjects. In other words, MLR does not require time-structured data; each subject in the data set can be assessed at a different number of measurement occasions with randomly assigned temporal spacing. In this case, the explanatory variable 'time' just takes on different values for each subject.

The distinction between MLR and LGC analysis is now blurring. Advanced structural equation modeling software is now incorporating some multilevel features. The latest version of M*plus*, for example, goes a long way towards bridging the gap between the two approaches (see e.g. Muthén, 2000; Muthén & Muthén, 2001). On the other hand, MLR software is incorporating features of LGC modeling. Two MLR software packages allow linear relations between the growth parameters (HLM, and GLLAMM), and GLLAMM allows the estimation of factor loadings. In addition, S-PLUS, HLM, and PRELIS as part of the LISREL 8.53 program offer a variety of residual covariance structures for MLR models. We believe that this tendency will continue, and that eventually the two approaches to growth curve modeling will merge into one another. However, until that time this paper may help to facilitate the choice between multilevel regression analysis and latent growth curve modeling for answering substantive questions.

References

Arbuckle, J.L. (1999). *Amos 4,* [Computer software]. Chicago: Smallwaters Corp.

Bandalos, D.L. (2002). The effects of item parceling on goodness-of-fit and parameter estimate bias. *Structural Equation Modeling, 9,* 78-102.

Bock, R. D. (1979). Univariate and multivariate analysis of variance of time-structured data. In J. R. &. B. P. B. Nesselroade (Eds.), *Longitudinal research in the study of behavior and development* (pp. 199-231). New York: Academic Press.

Bollen, K. A. (1989). *Structural equations with latent variables.* New York: Wiley.

Browne, W. J., & Rasbash, J. (2002). Multilevel Modelling. A. Bryman, & M. Hardy (Eds.), *To appear in: Handbook of Data Analysis.*

Bryk, A. S., & Raudenbush S.W. (1987). Application of hierarchical linear models to assessing change. *Psychological Bulletin, 101,* 147-158.

Bryk, A. S., & Raudenbush S.W. (1992). *Hierarchical Linear Models in Social and Behavioral Research: Applications and Data Analysis Methods.* Newbury Park, CA: Sage .

Bryk, A. S., & Raudenbush S.W. & Congdon, R. (1999).*HLM5: Hierarchical Linear and Nonlinear Modeling,* [Computer Software]. Chicago: Scientific Software.

Byrne, B. M., Shavelson, R. J., & Muthén, B. (1989). Testing for the equivalence of factor covariance and meanstructures: the issue of partial measurement invariance. *Psychological Bulletin, 105,* 456-466.

Chan, D., Ramey, S., Ramey, C., & Schmitt, N. (2000). Modeling intraindividual changes in children's social skills at home and at school: a multivariate latent growth approach to understanding between-settings differences in children's social skill development. *Multivariate Behavioral Research, 35,* 365-396.

Chou, C. P., Bentler, P. M., & Pentz, M. A. (1998). Comparison of two statistical approaches to study growth curves. *Structural Equation Modeling, 5,* 247-266.

De Leeuw, J., & Kreft, I. G. G. (1986). Random Coefficient Models for Multilevel Analysis. *Journal of Educational Statistics, 11,* 57-86.

Duncan, T. E., Duncan, S. C., Strycker, L. A., Li, F., & Alpert, A. (1999). *An introduction to latent growth curve modeling: Concepts, issues, and applications.* Mahwah, NJ: Lawrence Erlbaum Associates.

Eliason, S. R. (1993). *Maximum Likelihood Estimation: logic and practice.* Thousand oaks, CA: Sage.

Garst, H. (2000). *Longitudinal research using structural equation modeling applied in studies of determinants of psychological well-being and personal initiative in East Germany after the unification.* Unpublished doctoral dissertation, University of Amsterdam.

Garst, H., Frese, M., & Molenaar, P. C. M. (2000). The temporal factor of change in stressor-strain relationships: A growth curve model on a longitudinal study in East Germany. *Journal of Applied Psychology, 85,* 417-438.

Goldstein, H. (1986). Multilevel mixed linear model analysis using iterative generalised least squares. *Biometrika, 73,* 43-56.

Goldstein, H. (1987). *Multilevel Models in Educational and Social Research.* London, GB: Griffin.

Goldstein, H. (1995). *Multilevel statistical models.* London: Edward Arnold.

Hancock, G. R., Kuo, W. L., & Lawrence, F. R. (2001). An illustration of second-order latent growth models. *Structural Equation Modeling, 8,* 470-489.

Hox, J. J. (2000). Multilevel analysis of grouped and longitudinal data. In T. D. Little, K. U. Schnabel, & J. Baumert (Eds.), *Modeling longitudinal and multilevel data* (pp. 15-32). Mahwah: Lawrence Erlbaum.

Hox, J.J. (2002). *Multilevel anaysis: Techniques and applications.* Mahwah: Lawrence Erlbaum.

Hox, J. J., & Maas, C. J. M. (2001). The accuracy of multilevel structural equation modeling with pseudobalanced groups and small samples. *Structural Equation Modeling, 8,* 157-174.

Jöreskog, K. G., & Sörbom, D. (2002). *LISREL 8.52,* [Computer Software]. Chicago: Scientific Software.

Kenny, D. A., & Campbell, D. T. (1989). On the measurement of stability in over-time data. *Journal of Personality, 57,* 445-481.

Laird, N. M., & Ware, J. H. (1982). Random effects models for longitudinal data. *Biometrics, 38,* 963-974.

Li, F., Duncan, T. E., Duncan, S. C., McAuley, E., Chaumeton, N. R., & Harmer, P. (2001). Enhancing the psychological well-being of elderly individuals through Tai Chi exercise: A latent growth curve analysis. *Structural Equation Modeling, 8,* 53-83.

Li, F., Duncan, T.E., Harmer, P., Acock, A. & Stoolmiller, M. (1998). Analyzing measurement models of latent variables through multilevel confirmatory factor analysis and hierarchical modeling approaches. *Structural Equation Modeling, 5,* 294-306.

Littell, Milliken, Stroup and Wolfinger (1996) *SAS System for Mixed Models.* Cary, NC: SAS Institute.

Little, R. J. A., & Rubin, D. B. (1987). *Statistical analysis with missing data.* New York: Wiley.

Little, T. D., Schnabel, K. U., & Baumert, J. (Eds.). (2000). *Modeling longitudinal and multilevel data: practical issues, applied approaches, and specific examples.* Mahwah: Lawrence Erlbaum.

Loehlin, J. C. (1987). *Latent Variable Models.* Baltimore: Lawrence Erlbaum.

Longford, N. T. (1987). A fast scoring algorithm for maximum likelihood estimation in unbalanced mixed models with nested effects. *Biometrika, 74,* 817-827.

Longford, N. T. (1993). *Random coefficients models.* Oxford, UK: Clarendon.

MacCallum, R. C., Kim, C. (2000). Modeling multivariate change. In T. D. Little, K. U. Schnabel, & J. Baumert (Eds.), *Modeling longitudinal and multilevel data* (pp. 51-68). Mahwah: Lawrence Erlbaum.

MacCallum, R. C., Kim, C., Malarkey, W. B., & Kiecolt-Glaser, J. K. (1997). Studying multivariate change using multilevel models and latent growth curve models. *Multivariate Behavioral Research, 32,* 215-253.

McArdle, J. J. (1986). Latent variable growth within behavior genetic models. *Behavior Genetics, 16(1),* 163-200.

McArdle, J. J. (1988). Dynamic but structural equation modeling of repeated measures data. In R. B. Cattel, & J. Nesselroade (Eds.), *Handbook of multivariate experimental psychology* (2nd ed., pp. 561-614). New York: Plenum Press.

Meredith, W. M., & Tisak, J. (1990). Latent curve analysis. *Psychometrika, 55,* 107-122.

Mooney, C. Z. & Duval, R. D. (1993). *Bootstrapping: A nonparametric approach to statistical inference.* Newbury Park, CA: Sage.

Muthén, B. (1994). Multilevel covariance structure analysis. *Sociological Methods & Research, 22,* 376-398.

Muthén, B. (1997). Latent variable modeling of longitudinal and multilevel data. In A. Raftery (eds.), *Sociological Methodology* (pp. 453-480). Boston: Blackwell Publishers.

Muthén, B. (2000). Methodological issues in random coefficient growth modeling using a latent variable framework: Applications to the development of heavy drinking. In J. Rose, L. Chassin, C. Presson & J. Sherman (eds.), *Multivariate Applications in Substance use Research* (pp. 113-140). Hillsdale, N.J.: Erlbaum.

Muthén, B., Kaplan, D., & Hollis, M. (1987). On structural equation modeling with data that are not missing completely at random. *Psychometrika, 52,* 431-462.

Muthén, B., & Khoo, S. (1998). Longitudinal studies of achievement growth using latent variable modeling. *Learning and Individual Differences, 10,* 73-102.

Muthén, L. K. & Muthén, B.O. (1998). *Mplus 1.04* [Computer software]. Los Angeles: Muthén & Muthén.

Muthén, L.K., & Muthén, B.O. (2001). *Mplus: the comprehensive modeling program for applied researchers : user's guide.* Los Angeles: Muthén & Muthén.

Neale, M.C. (2002). *MX32,* [Computer software]. Richmond: Department of Psychiatry.

Oort, F. J. (2001). Three-mode models for multivariate longitudinal data. *British Journal of Mathematical and Statistical Psychology, 54,* 49-78.

Plewis, I. (1996). Statistical methods for understanding cognitive growth: a review, a synthesis and an application. *British Journal of Mathematical and Statistical Psychology, 49,* 25-42.

Plewis, I. (2000). Evaluating Educational Interventions Using Multilevel Growth Curves: The Case of Reading Recovery. *Educational Research and Evaluation, 6,* 83-101.

Rabe-Hesketh, S., Pickles, A. and Skrondal, A. (2001). *GLLAMM Manual.* Technical Report 2001, Department of Biostatistics and Computing, Institute of Psychiatry, King's College, London.

Rao, C. R. (1958). Some statistical methods for comparison of growth curves. *Biometrics, 14,* 1-17.

Rasbash, J., Browne, W., Healy, M., Cameron, B. & Charlton, C. (2000). *MLwiN1.10* [Computer software]. London: Multilevel Models Project Institute of Education.

Raudenbush S.W., & Bryk, A. S. (2002). *Hierarchical Linear Models: Applications and Data Analysis Methods. Second Edition.* Thousand Oaks, CA: Sage.

Raudenbush S.W., & Chan, W. S. (1992). Growth curve analysis in accelerated longitudinal designs. *Journal of Research in Crime and Delinquency, 29,* 387-411.

Raudenbush S.W., & Chan, W. S. (1993). Application of a hierarchical linear model to the study of adolescent deviance in an overlapping cohort design. *Journal of Consulting and Clinical Psychology, 6,* 941-951.

Raudenbush S.W., Rowan B., & Kang S.J. (1991). A Multilevel, Multivariate Model for Studying School Climate with Estimation via the EM Algorithm and Application to U.S. High-School Data. *Journal of Educational Statistics , 16,* 295-330.

Rovine, M. J., & Molenaar, P. C. M. (1998). A nonstandard method for estimating a linear growth model in LISREL. *International Journal of Behavioral Development, 22,* 453-473.

Rovine, M. J., & Molenaar, P. C. M. (2000). A structural modeling approach to a multilevel random coefficients model. *Multivariate Behavioral Research, 35,* 51-88.

Singer, J.D. (1998). Using SAS PROC MIXED to fit multilevel models, hierarchical models, and individual growth curve models. *Journal of Educational and Behavioral Statistics, 24,* 323-355.

Snijders, T.A.B. (1996). Analysis of longitudinal data using the hierarchical linear model. *Quality and Quantity, 30,* 405-426.

Snijders, T. A. B., & Bosker, R. J. (1999). *Multilevel analysis: An introduction to basic and advanced multilevel modeling.* London: Sage.

Stoel, R.D., & Van den Wittenboer, G. (2003). Time dependence of growth parameters in latent growth curve models with time invariant covariates. *Methods of Psychological Research, 8,* 21-41.

Stoel, R.D., van den Wittenboer, G. & Hox, J.J. (2003). Methodological issues in the application of the latent growth curve model. To appear in K. van Montfort, H. Oud, and A. Satorra (Eds.). *Recent developments in structural equation modeling: Theory and applications.*

Tucker, L. R. (1958). Determination of parameters of a functional relation by factor analysis. *Psychometrika, 23,* 19-23.

Van der Leeden, R. (1998). Multilevel analysis of longitudinal data. In C. C. J. H. Bijleveld, & L. J. Th. Van der kamp (Eds.), *Longitudinal data analysis* (pp. 269-317). London: Sage.

Willett, J. B., & Sayer, A. G. (1996). Cross-domain analyses of change over time: combining growth modeling and covariance structure analysis. In G. A. Marcoulides, & R. E. Schumacker (Eds.), *Advanced structural equation modeling: Issues and techniques* (pp. 125-157). Mahwah, New Jersey: Erlbaum.

Willett, J. B., & Sayer, A. G. (1994). Using covariance structure analysis to detect correlates and predictors of individual change over time. *Psychological Bulletin, 116,* 363-3381.

Wothke, W. (2000). Longitudinal and multi-group modeling with missing data. In T. D. Little, K. U. Schnabel, & J. Baumert (Eds.), *Modeling longitudinal and multilevel data* (pp. 219-240). Mahwah, NJ: Lawrence Erlbaum Associates.

Appendix

Let there be four measurement occasions. Let d_t (for $T = 1, 2, 3, 4$) be dummy variables for the measurement occasions such that if $T = k$, $d_k = 1$ and $d_{t \neq k} = 0$. For example: if $T = 1$, $d_1 = 1$ and $d_2 = 0$, $d_3 = 0$ and $d_4 = 0$. Using interactions between the time-varying covariate with dummy variables for the occasions, it is possible to estimate different effects (π_{2t}) for the time-varying covariate at each occasion. Now, Equations 1 can be written as:

$$\begin{aligned}
y_{ti} &= \pi_{0i} + \pi_{1i} T_{ti} + \pi_{21} x_{1i} d_1 + \pi_{22} x_{2i} d_2 + \pi_{23} x_{3i} d_3 + \pi_{24} x_{4i} d_4 + e_{ti} \\
\pi_{0i} &= \beta_{00} + \beta_{01} z_i + r_{0i} \\
\pi_{1i} &= \beta_{10} + \beta_{11} z_i + r_{1i}
\end{aligned}$$

Of course, this model can be represented also with three dummies instead of four.