Comparing and Combining Different Approaches to the Multitrait-Multimethod Matrix

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Campbell and Fiske (1959) introduced the multitrait multimethod (MTMM) approach as a means for investigating the construct validity of measures. The universe of observations in the MTMM approach is defined by the Cartesian product of two facets: the trait facet and the method facet. Thus, each trait is measured with each method, which leads to a full factorial design. Campbell and Fiske distinguish between convergent validity and discriminant validity. Convergent validity requires close agreement between measures of the same construct made by different methods. Discriminant validity requires that different constructs can be differentiated even if they are measured using similar methods.

Campbell and Fiske proposed to arrange the correlations between measures in a MTMM correlation matrix. The MTMM matrix thus contains correlations between measures using the same methods, which are referred to as *mono-method correlations*, and correlations between traits measured using different methods, which are referred to as *hetero-method correlations*. Two types of mono-method correlations can be distinguished: correlations between measures of different traits measured by the same method, referred to as discriminant validity coefficients, and correlations between identical traits measured by the same method, referred to as reliabilities. Two types of hetero-method correlations can be distinguished: correlations between different methods for the same trait, referred to as convergent validity coefficients, and correlations between different traits measured by different methods, referred to as nonsense correlations. Based on these distinctions, Campbell and Fiske (1959) suggested that four criteria should hold if the measures possess convergent and/or discriminant validity. These criteria are:

1. The convergent validity coefficients must be statistically significant and high. Failure of this test implies that the different methods are measuring different traits, meaning a lack of convergent validity.

2. The convergent validity coefficients must be higher than the nonsense correlations in the same row and column in which the individual validity coefficient is located. Failure of this test implies a lack of discriminant validity.

3. The convergent validity coefficients must be higher than the off-diagonal correlations in the corresponding mono-method blocks. If the validity coefficients are not substantially higher, this suggests that the traits are highly correlated or that there is a strong method effect, or both.

4. All submatrices of intertrait correlations should have the same pattern, independent of the method used.

Compliance with these four criteria is in practice evaluated by comparing the size of correlation coefficients between and within traits and methods. The evaluation of the measures' validity is based on a visual inspection of the correlation matrix, and simply counting the number of times the criteria are violated (see also Schmidt & Stults, 1986).

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Analysis models for the MTMM matrix

Confirmatory Factor Analysis

One problem of the Campbell/Fiske criteria is that they are based on an inspection of the raw correlations, which can be attenuated by measurement error. Another problem is that they are rather informal. In addition to these informal criteria, a number of formal models have been proposed to describe MTMM matrices in the presence of measurement error. The prevalent approach is to use confirmatory factor analysis to model the correlations or covariances in the MTMM matrix. For an overview see Alwin (1974), Widaman (1985) and Saris and Andrews (1991). The basic MTMM confirmatory factor model for a 3×3 MTMM matrix is given in Figure 1 below.

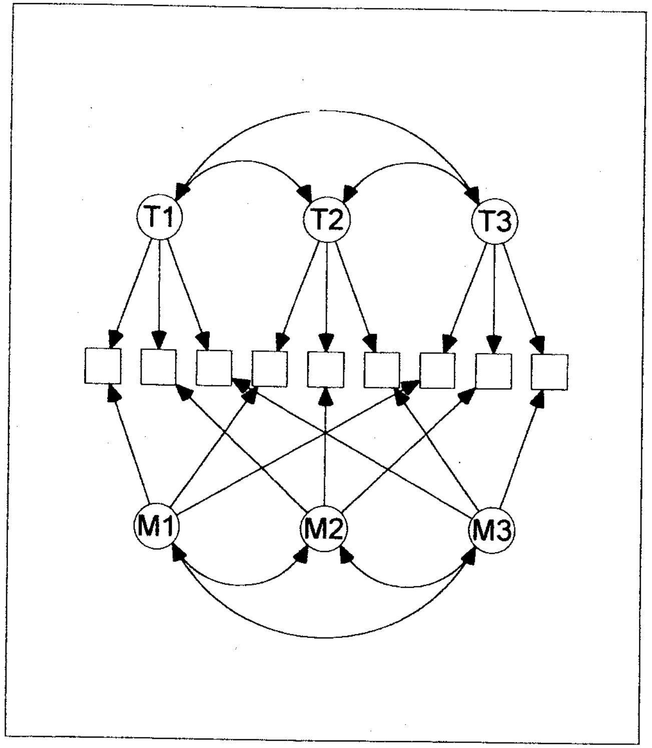


Figure 1. The basic confirmatory factor model for a 3×3 MTMM matrix.

Figure 1 implies that the factor matrix has a column for each trait and a column for each method. Each measure has one loading on the corresponding trait factor and one loading

on the corresponding trait factor. In the most general case there are no restrictions on the correlations between the latent factors (cf. Schmitt & Stults, 1986). However, this model often leads to unstable parameter estimates (Marsh, 1983). More usually the trait factors are assumed to be uncorrelated with the method factors, which is more stable and has the additional advantage that it leads to an orthogonal decomposition of the measures' variance into trait, method and residual error variance. Specific restrictions on the factor loading matrix or the factor correlation matrix can be used to test for special cases, such as zero correlations between method factors, equal method effects across different traits, and so on (cf. Widaman, 1985, for an overview and an analysis strategy).

The equation for the confirmative factor model, in the usual Lisrel notation (Bollen, 1989), is

$$\Sigma = \Lambda \Psi \Lambda' + \Theta_\delta \qquad (1).$$

In this equation, Λ is the factor loading matrix, Ψ the matrix of factor covariances or correlations, and Θ_δ is the vector of residual error variances. The parameters (factor loadings and intercorrelations) of the confirmatory factor analysis model have no direct relationship to the Campbell and Fiske criteria, but they can nevertheless be used to assess the convergent and discriminant validity of the measures. In general, factor intercorrelations can be interpreted as discriminant validity. The squared factor loadings provide an estimate of the variance related to trait, method, and residual error. The proportion of variance attributable to the trait can be interpreted as an upper limit for the measure's proportion true-score variance or its construct-related reliability.

Geometric Representation

Graphical representation of similarity data have for a long time been a mainstay of Facet Analysis (cf. Lingoes, Roskam & Borg, 1979). Campbell and Fiske discuss the MTMM in terms of the evaluation of the validity of measures, but the MTMM is also a special case of a factorial Facet Design with a trait and a method facet, and each structuple defining a measure (cf. Borg & Shye, 1995). A minimal mapping sentence for the basic MTMM has the form:

	(m1	*method1)*		*(t1*	*trait1)*
The	*(m2*	*method2)*	*assessment of*	*(t2*	*trait2)*
	(m3	*method3)*		*(t3*	*trait3)*

—————————————> *(response range)*

The MTMM correlation matrix can be analyzed using Smallest Space Analysis (SSA) or other geometric approaches to establish a low dimensional representation of the similarities in the MTMM matrix. Arguments for various ways in which the space can be partitioned can be found in reflecting on the order among elements in the trait and method facets. In general, Facet Theory does not focus on similarities and dimensions, but on partitions in the geometric space (Borg & Shye, 1995). In general, the traits will be qualitative or nominal; no particular order is assigned to the trait facet. Assuming valid measures, the Campbell and Fiske criteria imply that the points representing identical traits should be close to each other, and well separated from the points representing different traits. Thus, valid measures in a MTMM design should partition the space into regions that correspond to the traits, with the points representing identical traits preferably concentrated in clearly separated clusters. On the other hand, the method facet is probably more often assigned an (empirical) order. One purpose of the MTMM approach is to identify valid methods, that is, methods that produce little specific method variance. Methods that produce much method variance will increase the correlation between measures that share that method. The average correlation of variables is higher in the central region of the geometric space. As a consequence, if the MTMM matrix under consideration contains methods that differ in the amount of method variance they produce, this will show up in a radial partitioning of a two-dimensional representation.

The Composite Direct Product Model

The confirmatory factor analysis approach to the MTMM matrix suffers from a number of problems. To be identified, it requires at least three traits and three methods. The parameter estimates are often improper (e.g., negative variances) and contradictory to the patterns observed in the correlation matrix. Such problems may be the result of problems in the observed correlation matrix, but more probable causes are empirical under-identification, misspecification, and estimating too many parameters with too few measures (for an overview of these and other issues see Wothke, 1995). An alternative for the analysis of MTMM matrices with covariance structure models is the direct product model (Browne, 1984), given by:

$$\Sigma = D_x(\Pi_\mu \otimes \Pi_\tau + E)D_x \qquad (2).$$

In this equation, Π_μ and Π_τ are respectively the method and trait correlation matrices of the multiplicative factors, **E** is a diagonal matrix of residual errors, and **D** is a diagonal matrix of scaling constants to make the model describe a correlation matrix. The model expresses the observed correlations corrected for attenuation as a multiplicative function of the trait and method correlations in Π_μ and Π_τ. The discriminant validity coefficients can be found in the trait intercorrelations which are the elements of Π_τ. Convergent validity is maximal when the off-diagonal elements in Π_μ approach unity (cf. Wothke, 1995).

The direct product model has the advantage that it is identified with fewer than three traits and methods, and that its parameter estimates are more stable. In additional, Campbell and O'Connell (1982) showed that many observed MTMM matrices appear to have a multiplicative rather than an additive structure.

Since the direct product model produces generally stable estimates for the disattenuated correlations between traits and methods, an interesting approach is to apply SSA or similar analysis methods to the disattenuated facet correlations in Π_μ and Π_τ. This makes it possible to investigate geometrical patterns for separate facets, partialing out both error variance and the variance from other content facets. Because of its inherent instabilities, the confirmatory factor model is less suitable for this approach.

An Empirical Example

The data of the example consist of the responses of 473 respondents to 25 questions in a 5×5 MTMM matrix from a study on well-being (Hox, 1986). The MTMM contains five traits: global happiness, global satisfaction, and satisfaction about income, housing and health. There are also five methods: five question types denoted as standard, social comparison, ladder, faces, and circles. An interesting feature of these MTMM data is that they show positive manifold: all covariances in the covariance matrix are positive. Thus, one criterium for a plausible model is that all estimates must be positive as well.

Confirmatory Factor Analysis

For these data, the basic MTMM model indeed runs into the kind of problems described above (for details see Hox, 1995). The best estimation method is robust GLS estimation. This produces a robust χ^2 of 287 on 230 degrees of freedom (p=.01) and goodness-of-fit indices of 1.00 (GFI, AGFI, IFI). The factor matrix is given in table 1.

Table 1. Factor loadings and standard errors (GLS, decimal point omitted).

Var.	Method Factors					Trait factors				
	I	II	III	IV	V	I	II	III	IV	V
1	65 (05)					43 (06)				
2	56 (05)						67 (05)			
3	32 (04)							76 (04)		
4	28 (04)								74 (04)	
5	27 (04)									79 (04)
6		50 (05)				52 (05)				
7		65 (05)					44 (05)			
8		37 (05)						55 (04)		
9		29 (04)							63 (04)	
10		37 (04)								66 (04)
11			59 (06)			67 (06)				
12			61 (05)				69 (05)			
13			24 (04)					87 (04)		
14			26 (04)						83 (04)	
15			21 (03)							83 (04)
16				28 (08)		83 (05)				
17				29 (04)			81 (05)			
18				28 (04)				87 (04)		
19				28 (04)					86 (04)	
20				21 (04)						87 (04)
21					56 (06)	68 (06)				
22					49 (06)		75 (05)			
23					28 (04)			86 (04)		
24					30 (04)				80 (04)	
25					26 (04)					83 (04)

The factor loadings in table 1 show that trait factors I (general happiness) and II (general satisfaction) are not measured very well, which points to a relatively low convergent validity. The specific well-being factors III (income), IV (health) and V (housing) are measured with high accuracy. The fourth measure (happy/unhappy faces) has the least amount of specific measurement variance. The correlation between the trait factors is generally low (around .40), with the exception of general happiness and general satisfaction; these correlate 0.83. Thus the two traits general happiness and general satisfaction also show a lack of discriminant validity.

Geometric Representation

Figure 2 on the next page shows a 2-dimensional coordinate mapping of the 25 questions, with symbols representing the 5 traits. It is clear that the specific satisfaction traits are well separated, while the global happiness and satisfaction traits are not. It is also clear that the space can be partitioned into four regions, one for each specific satisfaction, and one for global well-being (happiness+satisfaction).

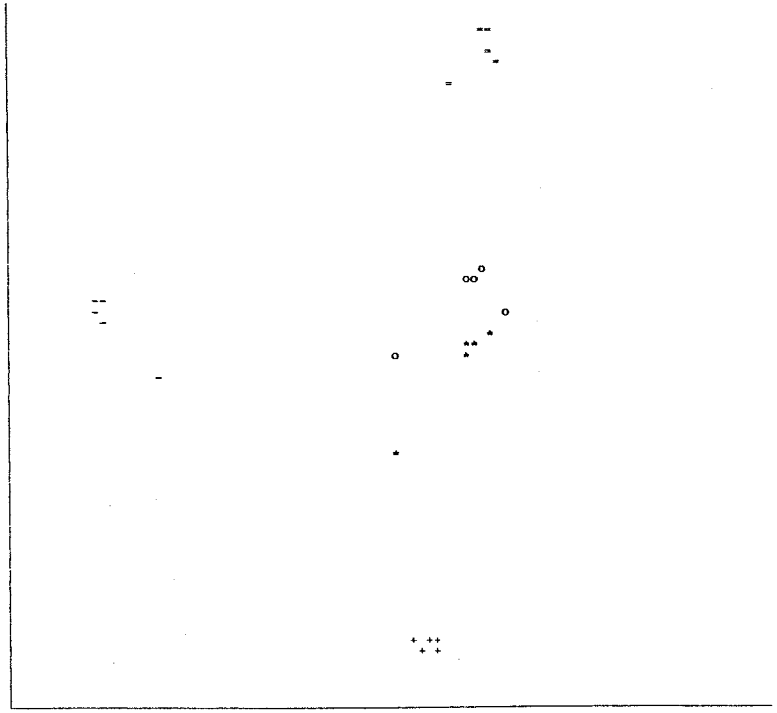


Figure 2. Two-dimensional mapping of well-being MTMM items.
Symbols: * satisfaction; **0** happiness; - house; + income; = health

Figure 3 below shows the same mapping, but now the symbols denote the different methods. There is no well-defined pattern in this mapping. There is some tendency for method *b* to be in the center of the space, meaning that method *b* (social comparison question) has a relatively high specific method effect.

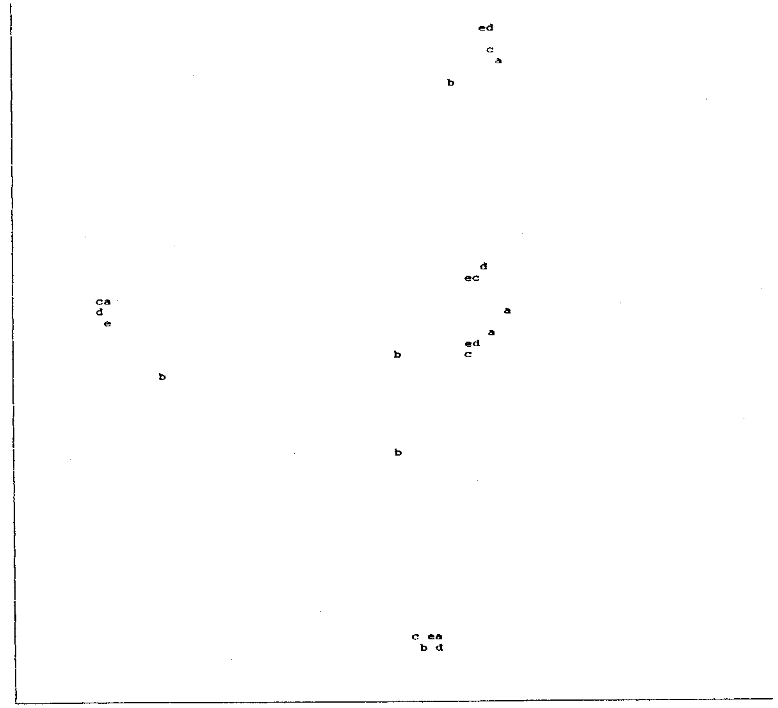


Figure 3. Two-dimensional mapping of well-being MTMM items.
Symbols: **a** standard question; **b** social comparison; **c** ladder; **d** faces; **e** circles

The Composite Direct Product Model

The composite direct product model assumes a multiplicative rather than an additive structure. Cudeck (1988) describes an informal test for the relation between hetero-method and mono-method correlations, based on comparing two loss-functions ϕ_1 and ϕ_2 for different blocks of the MTMM matrix. For these data, this procedure favors the multiplicative model (most tests in favor of the multiplicative model, some undecided). Table 2 below presents the disattenuated correlations between trait and method factors in the direct product model.

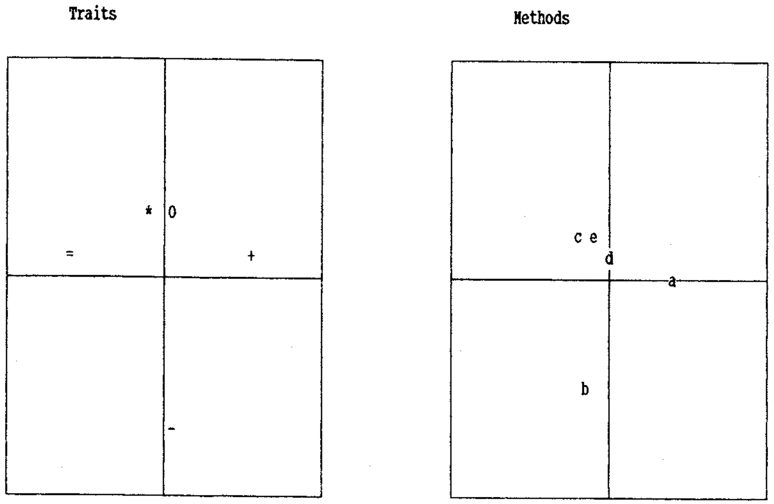
Table 2. Correlations between traits and methods, direct product model.

Traits	I	II	III	IV	V	Methods	I	II	III	IV	V
I	-						-				
II	89	-					62	-			
III	35	30	-				77	62	-		
IV	57	42	31	-			83	65	85	-	
V	51	51	19	26	-		79	62	83	88	-

The trait correlations show again that the general happiness and satisfaction factors correlate very high, implying low discriminant validity. In the ideal case, the methods matrix should contain all ones. Here, the three graphic question formats (ladder, faces, and circles) are clearly the best, with the social comparison question the worst, and the standard question in between.

Geometric Representation of the Direct Product Model Results

It is, of course, possible to produce a 2-dimensional mapping of the disattenuated trait and method correlations in Table 2. Figure 4 below shows the mapping for the five traits.



Symbols: * satisfaction, 0 happiness, - house, + income, = health

Symbols: a standard question, b social comparison, c ladder, d faces, e circles.

Figure 4. Two-dimensional plots of trait and method matrices in direct product model.

The plots on the previous page have a very straightforward interpretation. The traits plot shows again that the specific trait factors separate very well, while the global traits factors are close together and also closer to the center of the plot. Especially satisfaction with housing shows a very high discriminant validity. In the direct product model, those methods have the highest convergent validity which have the highest correlations in the methods correlation matrix. Those methods should also be the most central in the methods plot. Thus, the methods plot shows that method *b* (social comparison question) is the worst on this criterium, and method *d* (faces) the best. The other two graphical methods *c* (ladder) and *e* (circles) also perform well, and method *a* (standard question) is somewhere in between.

Discussion

The various methods to evaluate the validity of measures in a multitrait-multimethod design do in general lead to similar results. This is in itself reassuring, if only because it represents a case of convergent validity of analysis methods. There are some noteworthy differences also. The confirmatory factor model, which appears to work fairly well in this presentation, actually was rather difficult to manage. Different estimation methods had to be tried (not reported here, see Hox, 1995 for details) before a satisfactory solution could be found. The instability of the confirmatory factor model for these data is of course disturbing, and reflects negatively on its utility in general.

The geometric representation of the raw correlation matrices is by itself informative. It shows that the specific traits separate well, and that the global traits do not. It does not give so much information about the methods. A problem with an analysis of the raw correlations is that they may be attenuated by unreliability. Since the variables are single questions, it is not possible to use a reliability coefficient such as coefficient alpha to disattenuate the correlations.

The direct product model produces trait and method correlation matrices which can be directly inspected and interpreted. Nevertheless, it is surprising how much detail is gained if we inspect a geometrical representation of these correlation matrices. Especially the relative merit of the methods can be simply read from the methods plot.

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